Controllability of Complex Networks

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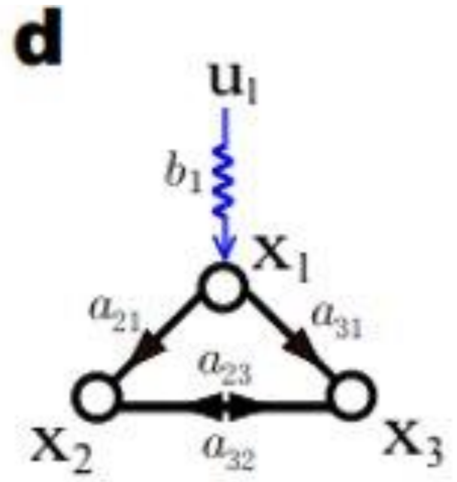
Controllability of Complex Networks

Yang-Yu Liu, Jean-Kacques Slotine, Albert-Lászlo´ Barabási (2011)

A dynamical system is controllable if, with a suitable choice of inputs, it can be driven from any initial state to any desired final state within finite time.

- N_D = minimum number of driver nodes required to control a network
- Kalman's controllability rank condition: $\mathrm{rank}(C) = N$



System A

$$\begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

Linear dynamics of system A

$$\mathbf{C} = b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & a_{23}a_{31} \\ 0 & a_{31} & a_{32}a_{21} \end{bmatrix}$$

Controllability Matrix of system A

$$C = (B, AB, A^2B, \ldots, A^{N-1}B)$$

Controllability of Complex Networks

- Minimum inputs theorem
  - minimum number of driver nodes relates to the size of a maximum matching
  - Perfect matching: $N_D \geq 1$
  - Imperfect matching: $N_D \geq |unmatched\ nodes|$
- Hub hypothesis: Control of hubs (nodes with large degree) is essential to controlling a network?
  - Nodes divided into three groups according to *k* (low, medium, high)
- Topological features determining network controllability
  - Real networks randomized with *N* and *L* unchanged
  - Real networks randomized with degree-preserving algorithm

The Fundamental Advantages of Temporal Networks

Aming Li, Sean P. Cornelius, Yang-Yu Liu, Long Wang, Albert-László Barabási

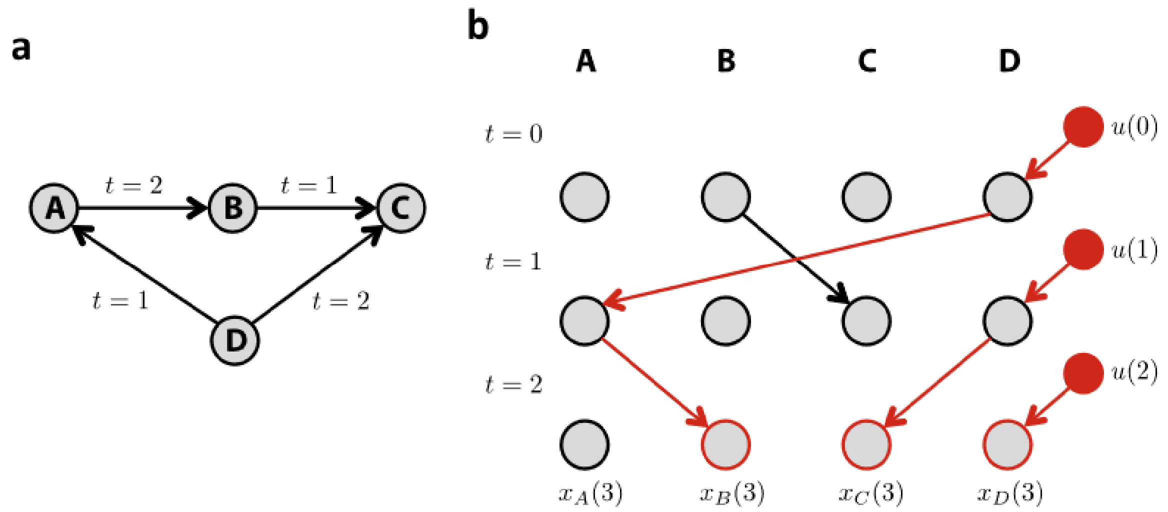
(2017)

- Traditional focus of network science is on static networks
- Advantages of adding temporality:
  - Temporal networks require less energy to bring to some final state
  - Temporal networks may reconcile the tradeoff between number of driver nodes and the time to control
- Motivation to study temporal networks

Structural Controllability of Temporal Networks

Márton Pósfai and Philipp Hövel (2014)

- Controllability is characterized as the average maximum controllable subspace over possible starting (intervention) nodes (Equation 1)



$$N_C(t, \Delta t) = \frac{1}{N} \sum_{v \in V} N_C(v, t, \Delta t).$$

Equation 1: Average maximum controllable subspace over possible intervention points in V

- Maximum controllable subspace is determined by the maximum number of independent paths starting from possible intervention points and ending at time *t*

Goals

- Extend techniques presented in "Controllability of Complex Networks" to temporal networks

- Explore the maximum controllable subspace as characterized in Structural Controllability of Temporal Networks on driver nodes vs non-driver nodes

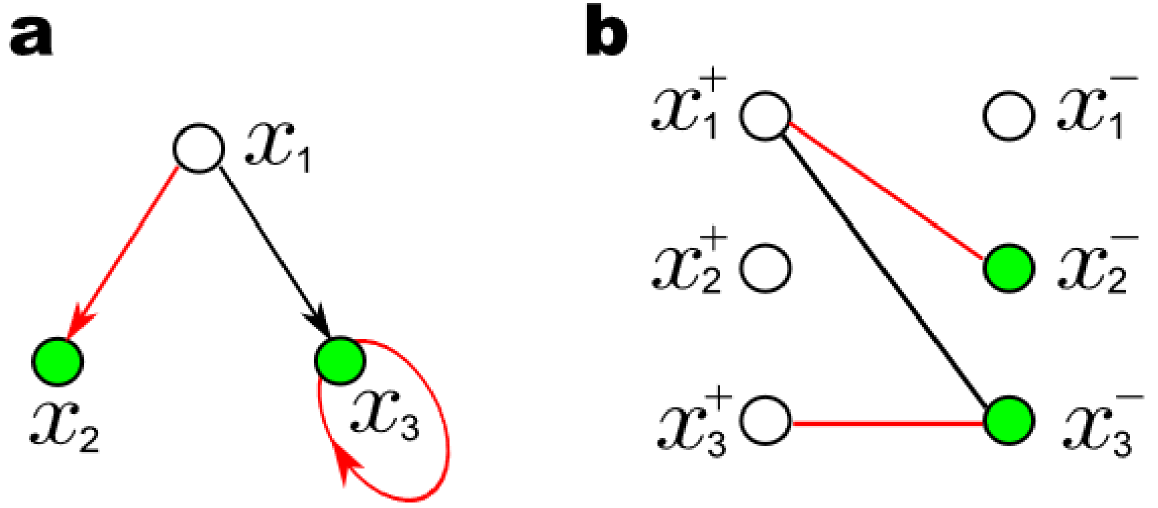
Datasets

- CollegeMsg (SNAP dataset)
  - Private messages sent on an online social network at the University of California, Irvine
  - 193 day timespan
- Dynamic Face-to-Face Interaction Network
  - Interactions (who-looks-at-whom) between *The Resistance* players
  - Data spans 62 games of 45-60 minutes each; a game spanning 53 minutes is used
- Ants
  - Interactions between ants in a colony of population 88
  - Data is annotated from a 24-minute video recording

- Choosing appropriate time intervals
  - How often/how long are edges active?
  - In what time period do we aim to control the network?
  - Smaller intervals = Sparser networks

Results: Maximum Matching ≈ N_D

- Perfect matching: $N_D \geq 1$; any random node can be chosen as the driver node
- Imperfect matching: $N_D \geq |unmatched\ nodes|$; unmatched nodes are chosen as driver nodes

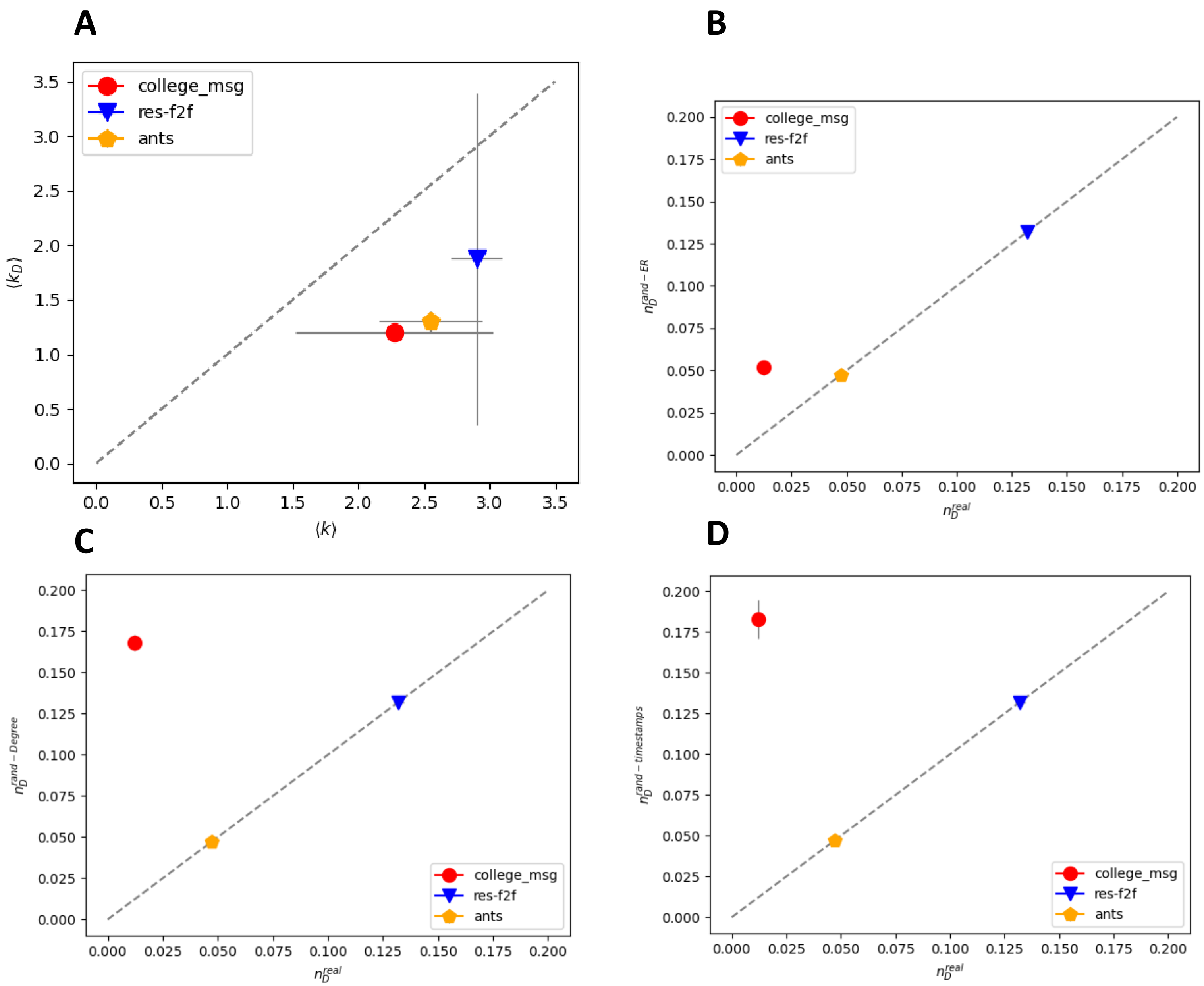


Name	N	L	Avg(*k*)	Avg(n_D^{real})	Avg($n_D^{rand\text{-}Degree}$)	Avg($n_D^{rand\text{-}ER}$)	Avg($n_D^{rand\text{-}t}$)	Time
CollegeMsg	1889	59834	0.165	0.01217	0.168	0.05183	0.183	192 days
Res-f2f	9	41	0.086	0.132	0.132	0.132	0.132	53 minutes
Ants	88	1852	0.877	0.0473	0.0473	0.0473	0.0473	24 minutes

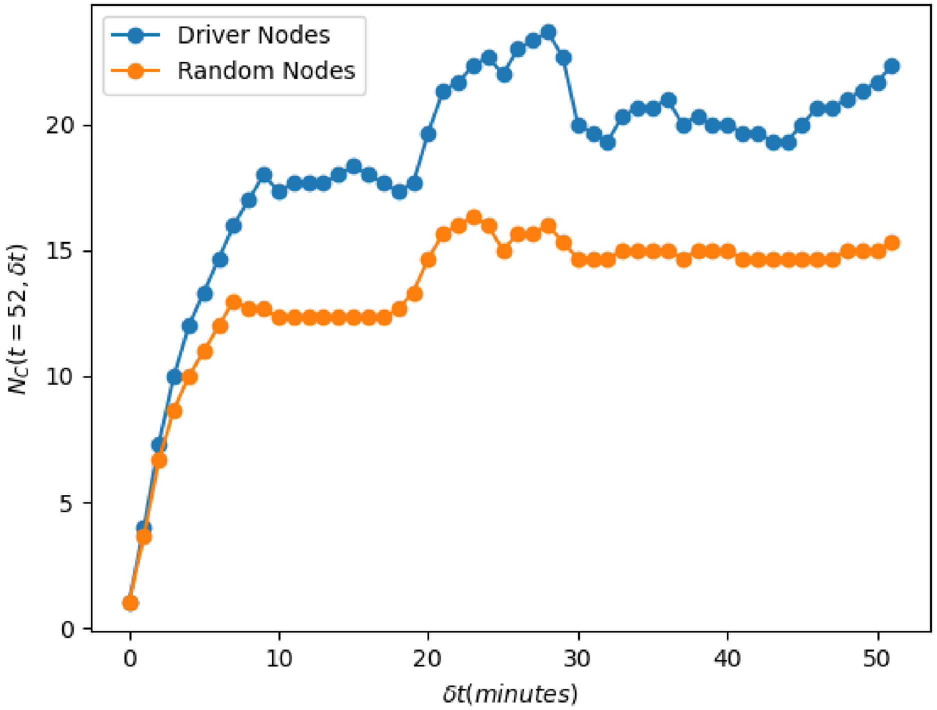
Results: Characterizing and Predicting Driver Nodes

A. The mean degree of driver nodes compared with the mean degree of all nodes.

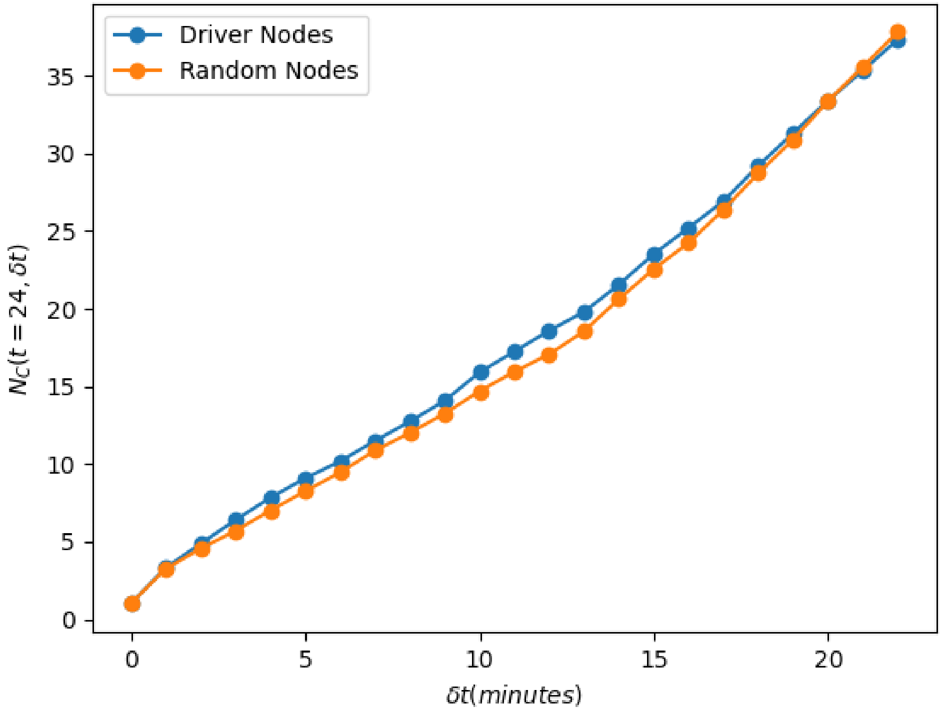
B-D. The number of driver nodes obtained for $n_D^{rand\text{-}ER}$, $n_D^{rand\text{-}Degree}$, and $n_D^{timestamp}$ respectively, compared to the number of driver nodes obtained from n_D^{real}.



Results: Average Maximum Controllable Subspace



A: Maximum controllable subspace over one The Resistance game



B: Maximum controllable subspace over an ant colony in a 24-minute time period

Conclusions

- The density of driver nodes is lower among hubs.
- N_D is controlled mostly by a graph's degree distribution.
  - Controllability is largely encoded by a graph's degree distribution

- The average maximum controllable subspace is higher when intervention points coincide with driver nodes.

Future Work

- Explore the significance of the effects of a node's criticality, as defined in "Controllability of Complex Networks", on the average controllable subspace.
- Focus on either one network, or a larger survey of networks to better understand behaviors based on graph topology.
    - Test how randomization procedures affect the controllability of a network through time, as seen in "Structural Controllability of Complex Networks"

Sources

- Liu, Yang-Yu, Jean-Jacques Slotine, and Albert-László Barabási. "Controllability of complex networks." *nature* 473.7346 (2011): 167-173.
- Pósfai, Márton, and Philipp Hövel. "Structural controllability of temporal networks." *New Journal of Physics* 16.12 (2014): 123055.
- Li, Aming, et al. "The fundamental advantages of temporal networks." *Science* 358.6366 (2017): 1042-1046.